QUANTUM PHYSICS I - Nov. 2, 2017

Write your name and student number on **all** answer sheets. There are four problems in this exam. You can earn 90 points in total, with exam grade equal to 1 + points/10.

PROBLEM 1: INFINITE POTENTIAL WELL (all 5 points)

The infinite potential well has zero potential energy between 0 and a, and is infinite elsewhere.

a) What are the energy eigenstates of this quantum system, and what are their energies? In the case of a discrete spectrum, explain where the quantization comes from.

b) Suppose we take the wavefunction at a given time to be an arbitrary function of x that is symmetric around the center of the well (at $x = a/2$). Is this a stationary state in general? Briefly explain your answer.

c) Start with the wavefunction of b), measure the location, and then wait for an hour and measure the energy of the state. Which outcomes are possible?

d) Suppose we start with the particle in its groundstate. At some point the well doubles in size, and extends from 0 to $2a$ (while the wavefunction is still given by the groundstate of the smaller well). What are the possible outcomes of an energy measurement of this system?

PROBLEM 2: COMPATIBLE OPERATORS (all 5 points)

Consider two operators acting on an infinite-dimensional Hilbert space.

a) When are two operators compatible? Briefly explain the implications of being (in-)compatible.

b) Are position and linear momentum in three spatial dimensions compatible operators? Briefly explain your anwer.

c) Are linear momentum and angular momentum in three spatial dimensions compatible operators? Briefly explain your anwer.

PROBLEM 3: ENTANGLED STATES (all 5 points)

Consider a two-fermion system of which we will only consider the orientation of the spin of the fermions. Both fermions have spin-1/2.

a) What is the notion of an entangled state for this system?

b) Is the state $\frac{1}{\sqrt{2}}(| \downarrow\uparrow> + | \uparrow\downarrow>)$ an entangled state? Explain your anwer.

c) In the Einstein-Podolsky-Rosen experiment, an electron-positron pair can be prepared in an entangled state. Briefly describe how the measurement of the spin orientation of the electron influences the measurement of the spin orientation of the positron, and vice versa. Can this be used to transmit information? Indicate whether quantum mechanics preserves locality and/or causality.

PROBLEM 4: BOHR MODEL (5 points each, b and c 10 points)

Consider the groundstate of the electron in the Bohr model of 3D hydrogen atom. We will take its wavefunction to have the form

$$\psi(r,\theta,\phi) = R(r)Y(\theta,\phi)\,, \qquad R(r) = \frac{2}{a^{3/2}}\exp(-r/a)\,,$$

in terms of the Bohr radius a.

a) What is the dependence on the angles θ and ϕ for the groundstate?

b) The Laplacian operator $\vec{\nabla}^2$ has a radial part given by

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)\,.$$

Derive the energy of the ground state in terms of a, $\hbar$ and the mass of the particle.

c) Derive the form of the potential energy in terms of a, $\hbar$ and the mass of the particle. Use that the ground state satisfies the Schrodinger equation.

d) Calculate the expectation value of r for the ground state.

e) Which quantum numbers characterize the first excited state of hydrogen, and which values can they take?

f) What can you say about the expectation values for the operator r evaluated on the different excited states - are these equal or smaller / larger, and how do they compare to the ground state? Briefly explain your answer.